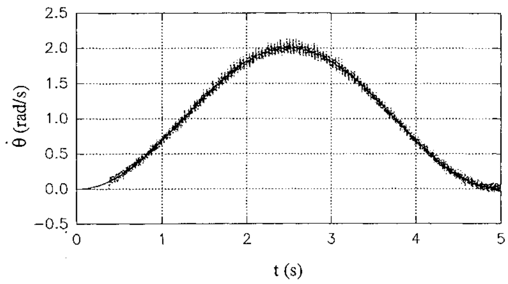

Fig. 2 Controlled elastic response.

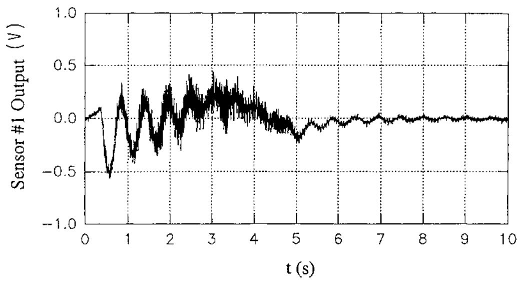

Fig. 3 Time history of angular velocity.

law [Eq. (9)], the disturbance counteracting control part is tuned manually to achieve best performance.

V. Summary and Conclusions

In this Note, the equations of motion are derived for the slewing active beam equipped with multitudes of piezoelectric sensors and actuators by means of the extended Hamilton principle. Based on these equations of motion, a decentralized control technique is proposed to meet the mission requirement. Hence, controls are divided into the slewing control and the vibration suppression controls. The sliding-mode control is proposed as the slewing control and the modal-space positive position feedback plus disturbance-counteracting control is developed for the suppression of vibrations.

The developed control techniques are applied to the testbed and verified by the experiment. The experimental results show that the decentralized controls performed satisfactorily but high-frequency vibrations remained uncontrolled. In order to maintain the control authority over these higher modes, more actuators and sensors are needed. In addition, more powerful piezoelectric actuators are desirable.

Acknowledgment

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Low-Thrust Orbit Transfer Guidance Using an Inverse Dynamics Approach

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Introduction

MUCH research has been devoted to computing optimal low-thrust trajectories for solar electric propulsion (SEP) spacecraft. By comparison, the volume of work on guidance laws for electric propulsion spacecraft is somewhat limited. Trajectory optimization methods often utilize optimal control theory and the resulting Euler-Lagrange or costate equations define the thrust vector steering during the optimal transfer in an open-loop fashion. For realistic onboard guidance schemes, this technique is not feasible for the very long duration transfers performed by low-thrust spacecraft. Early studies on low-thrust guidance involved optimal neighboring control techniques[1,2] and nonoptimum, feasible schemes using reference trajectories.[3] Recently, a low-thrust guidance method using Lyapunov stability theory has been proposed.[4]

In this Note, a new guidance scheme using an inverse dynamics approach is developed for a general, three-dimensional, low-thrust transfer from circular low Earth orbit (LEO) to geosynchronous orbit (GEO). The goal is to devise an autonomous, reliable guidance scheme that computes the necessary thrust-direction steering for the LEO-GEO transfer and results in near-optimal performance. The guidance scheme utilizes the optimal, minimum-fuel transfer as a reference trajectory. Numerical results are presented for a typical SEP vehicle designed for a 200-day LEO-GEO mission.

Inverse Dynamics Guidance

The inverse dynamics guidance scheme is based on the optimal time histories of the semimajor axis a, eccentricity e, and inclination i. Therefore, the governing differential equations for these orbital elements are required. The equations of motion in an inverse-square gravity field for these orbital elements are

$$\frac{\mathrm{d}a}{\mathrm{d}t} = \frac{2a^2 v}{\mu} a_T \cos\phi \cos\sigma \tag{1}$$

$$\frac{\mathrm{d}e}{\mathrm{d}t} = \frac{a_T}{v}\left[2(e + \cos\nu)\cos\phi\cos\sigma + \frac{r}{a}\sin\nu\sin\phi\cos\sigma\right] \tag{2}$$

$$\frac{\mathrm{d}i}{\mathrm{d}t} = \frac{r}{h}\cos(\omega + \nu) a_T \sin\sigma \tag{3}$$

In these equations, v is the velocity, r is the radial position, h is the angular momentum, ν is the true anomaly, ω is the argument of perigee, μ is the gravitational constant, a_T is the thrust acceleration, ϕ is the in-plane steering angle measured from the velocity vector to the projection of the thrust vector onto the orbit plane, and σ is the out-of-plane steering angle.

The guidance law is based on an inverse dynamics approach. Lu[5] recently presented an inverse dynamics approach to trajectory

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optimization for the National Aerospace Plane. The necessary control to track a given reference trajectory for an inverse dynamics problem can be determined from the governing equations of motion and repeated differentiation of the desired output time history. The reference trajectory is the minimum-fuel, LEO-GEO transfer as computed by the multiple-shooting method, low-thrust trajectory optimization code SECKSPOT.[6] Two constraint equations that enforce tracking of the reference trajectory are defined:

$$e(a) - e^*(a) = 0, \qquad a_0 \le a \le a_{\text{GEO}} \tag{4}$$

$$i(a) - i^*(a) = 0, \qquad a_0 \le a \le a_{\text{GEO}} \tag{5}$$

In Eqs. (4) and (5), $e^*(a)$ and $i^*(a)$ represent the optimal eccentricity and inclination histories with the optimal semimajor axis as the independent variable. The desired initial and final orbits are indicated by the limits on the semimajor axis, a_0 and a_{GEO}, respectively. Differentiating the constraints with respect to the semimajor axis results in

$$\frac{de}{da} - \frac{de^*}{da} = 0 \tag{6}$$

$$\frac{di}{da} - \frac{di^*}{da} = 0 \tag{7}$$

where de/da and di/da are obtained by dividing the time rate of change differential equations for e and i [Eqs. (2) and (3)] by the time rate differential equation for a [Eq. (1)]. The resulting differential equations with a as the independent variable are

$$\frac{de}{da} = \frac{\mu[r \sin\nu \sin\phi + 2a(\cos\nu + e)\cos\phi]}{2a^3 v^2 \cos\phi} \tag{8}$$

$$\frac{di}{da} = \frac{\mu r \cos(\omega + \nu) \sin\sigma}{2a^2 v h \cos\phi \cos\sigma} \tag{9}$$

Although we may now obtain the controls ϕ and σ using Eqs. (6–9), this is not desired since the equations of motion are highly oscillatory over long time durations due to the $\sin\nu$ and $\cos\nu$ terms. Also, the reference rates de^*/da and di^*/da are the averaged rates of change of the orbital elements due to the solution method of SECKSPOT. Therefore, the new differential constraint equations are

$$\frac{d\bar{e}}{d\bar{a}} - \frac{de^*}{da} = 0 \tag{10}$$

$$\frac{d\bar{i}}{d\bar{a}} - \frac{di^*}{da} = 0 \tag{11}$$

where $d\bar{e}/d\bar{a}$ and $d\bar{i}/d\bar{a}$ are the averaged rates of change for the orbital elements. These rates are computed through orbital averaging by calculating the incremental change in an orbital element during a single orbit and dividing by the period. The averaged time rates of change for the orbital elements $z = [a, e, i]^T$ are

$$\frac{d\bar{z}}{dt} = \frac{\Delta z}{\Delta t} = \frac{1}{T}\int_{\nu_i}^{\nu_f} \frac{dz}{dt}\frac{dt}{d\nu} d\nu \tag{12}$$

The integral represents the incremental change in an orbital element during one revolution with all orbital elements held constant except for true anomaly ν. The averaged rate is calculated by dividing the incremental change by the orbital period T. The true anomalies ν_i and ν_f represent the exit and entrance into Earth's shadow. The averaged rates with respect to the semimajor axis are computed by dividing $d\bar{e}/dt$ and $d\bar{i}/dt$ by $d\bar{a}/dt$.

In order to compute the averaged orbital rates, a steering parameterization must be included in the orbital element differential equations (dz/dt) in the integrand of Eq. (12). A simple guidance steering law based on true anomaly is implemented for ϕ and σ:

$$\phi = u_{\max} \sin\nu - \gamma \tag{13}$$

$$\sigma = -\sigma_{\max} \cos(\omega + \nu) \tag{14}$$

The steering angle u is measured from the local horizon to the in-plane thrust vector projection and is considered positive above the horizon in the direction of motion. The flight-path angle γ is measured in the orbital plane from the local horizon to the velocity vector. These steering laws are developed by observing that optimal steering solutions for low-thrust transfers exhibit a sinusoidal nature with varying amplitude.[7] In the case of in-plane steering, the optimal steering angle is zero at the apses and at maximum amplitude near ± 90 deg. For out-of-plane steering, a good steering strategy would follow $\cos(\omega+\nu)$ since this term dominates the differential equation for inclination.

Feedback is added to the inverse guidance scheme to improve tracking of the optimal trajectory. The simple feedback laws for the maximum amplitudes are

$$u_{\max} = u^*_{\max} - K_e(e - e^*), \qquad K_e > 0 \tag{15}$$

$$\sigma_{\max} = \sigma^*_{\max} + K_i(i - i^*), \qquad K_i > 0 \tag{16}$$

where $u^*_{\max}$ and $\sigma^*_{\max}$ are the values that produce the desired rates de^*/da and di^*/da after orbital averaging. This feedback law is implemented in the guidance algorithm to augment $u_{\max}$ and $\sigma_{\max}$. Steering is commanded between guidance cycles by the open-loop steering laws defined by Eqs. (12) and (14). A guidance cycle is defined as the time interval between calls to the inverse dynamics guidance algorithm. Therefore, the steering amplitudes $u_{\max}$ and $\sigma_{\max}$ are updated each guidance cycle.

The inverse dynamics guidance scheme can now be outlined. At each guidance cycle, the reference orbital elements e^* and i^* and element rates de^*/da and di^*/da are calculated for the current semimajor axis by cubic spline interpolation through 21 points with a as the independent variable. The averaged orbital element rates $d\bar{e}/d\bar{a}$ and $d\bar{i}/d\bar{a}$ are computed using Eq. (12) with steering determined by Eqs. (13) and (14) for an estimated $u_{\max}$ and $\sigma_{\max}$. Earth shadow exit and entrance true anomalies are computed and the integration over one revolution is computed via Gaussian quadrature. The steering amplitudes $u_{\max}$ and $\sigma_{\max}$ are adjusted by a Newton iteration until the desired reference rates are obtained. The converged amplitudes $u^*_{\max}$ and $\sigma^*_{\max}$ are augmented by the feedback terms and are then utilized in the steering law [Eqs. (13) and (14)] between guidance cycles in an open-loop fashion.

Numerical Results

The guidance law performance is demonstrated by a numerical simulation of the low-thrust transfer. The LEO-GEO mission begins at a 500-km-altitude circular Earth orbit with an inclination of 28.5 deg and terminates in a geosynchronous orbit with zero inclination. The initial spacecraft mass in LEO is 6100.8 kg, the I_{sp} is 3800 s, and the constant thrust magnitude is 2.25 N, which results in an initial thrust-to-weight (T/W) ratio of 3.8×10^{-5}. The spacecraft experiences total loss of power and therefore thrust in the Earth-shadowing periods during the orbital transfer.

The trajectory is simulated by numerically integrating the true, unaveraged, complete three-dimensional equations of motion in an equinoctial coordinate frame. The iterative guidance algorithm is initially called every 2.0 days to update the steering amplitudes $u_{\max}$ and $\sigma_{\max}$. The guidance cycle is decreased to 0.2 days after the semimajor axis reaches 3.0 Earth radii in order to improve tracking as the orbital period increases. The thrust vector steering is controlled by Eqs. (13) and (14) during the numerical integration of the trajectory. The values of the error gains K_e and K_i are 1000 deg and 50 deg/deg, respectively. These values were selected as the result of a few trial simulations. It was observed during these simulations that the guidance algorithm's performance is insensitive to the value of the gains since the tracking error is extremely small. All guidance updates to $u_{\max}$ and $\sigma_{\max}$ via orbital averaging converged in at most two iterations.

The guidance law based on inverse dynamics tracks the SECKSPOT optimal solution very nicely, as demonstrated in Fig. 1. The eccentricity history for the guided trajectory oscillates about the reference eccentricity profile. Each oscillation corresponds to one revolution during the transfer, and the frequency of oscillation

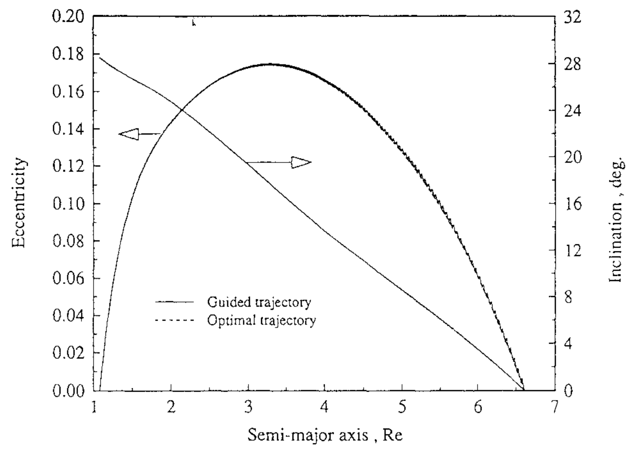

Fig. 1 Eccentricity and inclination vs semimajor axis.

decreases as the semimajor axis (and therefore orbital period) increases. Despite the inherent oscillatory behavior of eccentricity, the guidance law maintains good tracking of the averaged eccentricity profile for the minimum-fuel transfer. The reference inclination profile is tracked almost perfectly without error, as also demonstrated in Fig. 1. The final values of eccentricity and inclination at GEO altitude are 4×10^{-4} and 0.04 deg, respectively. The trip time and final mass are 201.1 days and 5220.6 kg. The optimal trajectory computed by SECKSPOT delivers 5239.5 kg to GEO in 197.8 days. Therefore, the guidance scheme requires 0.4% more fuel and 1.7% more time to complete the transfer. However, SECKSPOT approximates the entire trajectory, utilizing orbital averaging techniques, whereas the inverse dynamics guidance simulation involves accurate numerical integration of the unaveraged state equations.

The resulting in-plane steering amplitude u_{max} starts small (tangent steering), increases to maintain the positive eccentricity rate, and then becomes negative at the peak eccentricity. The steering amplitude must be negative after the peak eccentricity in order to shift the phase of the sinusoidal steering law and produce a negative eccentricity rate. The out-of-plane steering amplitude σ_{max} steadily increases during the transfer in order to complete the majority of the plane change near the end of the LEO-GEO maneuver.

Summary and Conclusions

A guidance scheme based on inverse dynamics has been devised for a three-dimensional, low-thrust transfer from LEO to GEO. The guidance problem is challenging since the transfer time is extremely long and coasting arcs are present during Earth-shadowing conditions. The guidance strategy utilizes the optimal minimum-fuel trajectory based on the averaged state equations as a reference trajectory. The guidance law uses orbital averaging coupled with a relatively simple steering law to provide accurate closed-loop tracking of the optimal trajectory. The orbit transfer guidance has proven quite effective in simulations performing this difficult maneuver in the context of the oscillatory unaveraged state equations and exhibits near-optimal performance. This guidance scheme could be implemented onboard a solar electric vehicle with modest computational and storage requirements and is therefore a viable candidate for an autonomous, low-thrust orbit transfer guidance system.

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System Design by Linear Exponential Quadratic Gaussian and Loop Transfer Recovery Methodology

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I. Introduction

A ROBUST feedback control system is a system that can satisfy the design requirements within the variation bounds of the actual system dynamics. In the past several years significant methods and advances have been made in integrating time-domain optimization-based control design techniques, such as linear quadratic Gaussian (LQG), with frequency-domain approaches. These integrated frequency-domain and state-space approaches to the design of multi-input, multi-output (MIMO) control system have culminated in the methodology called LQG with loop transfer recovery (LTR). The adoption of the LTR method is to improve the robustness of the LQG regulator with the state observer.[1,2]

In this Note, the linear exponential quadratic Gaussian (LEQG) performance criteria[3–6] and loop transfer recovery (LEQG/LTR) methodology for MIMO robust control system design are developed. It is known[3–6] that, for the LEQG problem, the separation principle can be applied, but the certainty equivalence principle cannot. Furthermore, it was shown[6] that optimal control of the LEQG problem could reduce the maximum and the standard deviation of the miss distance as well as the total rolling angle of a bank-to-turn (BTT) missile. This conclusion was based upon time-domain simulation without any frequency-domain interpretation. Therefore, the first purpose of this Note is to derive the algorithms for the MIMO robust control system design by the LEQG/LTR method, and the second purpose is to study the difference in frequency responses for systems designed by LEQG/LTR and LQG/LTR methodologies.

In this Note, it is shown that the LEQG/LTR method is similar to the LQG/LTR method but with a little modification; e.g., the LEQG/LTR method can also be applied to the design of return ratios at the output or input of the plant. For the former case, the first step is to design the Kalman filter, i.e., manipulating system and measurement noise covariances until a return ratio is obtained that is satisfactory at the plant output. The second step is to synthesize an optimal controller based on LEQG performance criteria. In addition to being able to manipulate the state and control weighting factors, an additional weighting factor can be adjusted to obtain better responses. However, this weighting factor is not included in the LQR/LTR method, causing the certainty equivalence principle to not be held by the LEQG/LTR method. Therefore, one can trade off σ to relax the recovery parameter without lowering the robustness (sometimes the robustness may even be in-

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